

THE INVERSE PROBLEM
IN
JET ACOUSTICS

Preliminary Version

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SUMMARY

The inverse problem for jet acoustics, or the determination of noise sources from far-field pressure information, is proposed as a tool for understanding the generation of noise by turbulence and for the improved prediction of jet noise. An idealized version of the problem is investigated first to establish the extent to which information about the noise sources may be determined from far-field pressure data and to determine how a well-posed inverse problem may be set up. Then a version of the industry-standard MGB code is used to predict a jet noise source spectrum from experimental noise data.

1. INTRODUCTION

The purpose of this paper is to describe an initial examination into the use of the inverse problem in acoustics to understand properties of turbulence-generated noise. That is, it is desired to use experimental determinations of the far-field pressure to generate information about the generation of noise by turbulent flows. Naturally, such an inverse problem must be carefully set up in order to give a mathematically well-posed problem. If this may be done successfully, however, there is a potential for eliciting important information about the still poorly understood process of the generation of noise by turbulence.

There has recently been a number of investigations into the use of sound waves as a diagnostic tool for fluid flows [1, 2, 3]. This technique involves passing sound waves through a flow and reconstructing the flow based on the changes in amplitude and phase in the sound waves produced by velocity variations in the flow. What is proposed here is somewhat different, both in technique and emphasis, because here we wish to learn about the nature of the sound sources rather than the moving fluid through which the sound propagates and the inversion is based on the sound waves produced by the flow itself, rather than sound waves applied externally.

The analysis of the inverse problem in jet acoustics discussed in this paper is based on the Lighthill acoustic analogy [4] and its refinements [5, 6]. We begin with an examination of the inverse problem in its simplest context, where the basic Lighthill acoustic analogy is used to relate far-field pressure information to the noise sources. This serves to establish how a well-posed inverse problem should be set up and what information we can and cannot expect to get from solving the inverse problem.

Having established the basic features of the inverse problem, we apply the MGB code of Khavaran [7] as modified in previous work [9] to permit arbitrary noise spectra to be

specified. Incorporating various corrections for convective and other effects, the MGB code provides a model for those aspects of the noise source that cannot be inferred from the far-field pressure data. We begin by attempting to determine the frequency distribution of the noise source from experimental data, with the remainder of the source information specified by the MGB code. Experimental data presented by Seiner *et al.* [10] is used in conjunction with aerodynamic data produced in earlier work [11] as inputs for the inverse problem solution procedure.

2. AN IDEALIZED INVERSE PROBLEM

In this section, the basic Lighthill acoustic analogy is used to relate noise sources to the far-field pressure spectrum in a relatively simple way. Idealized representations of the jet noise problem are used to determine how a well-posed inverse problem may be set up and to determine what information can be derived from the inverse problem and what information cannot. The experience gained through the examination of this idealized inverse problem will be applied to the formulation and numerical solution of the inverse problem for a jet flow in the following section.

Lighthill advanced an acoustic analogy [4] in 1952 for the analysis of flow-generated noise. He recast the Navier-Stokes equations as a wave equation for the acoustic pressure disturbance p that includes source terms involving the nonlinear convective terms,

$$\frac{1}{c_o^2} \frac{\partial^2 p}{\partial t^2} - \nabla^2 p = \frac{\partial T_{ij}}{\partial x_i \partial x_j} \quad (1)$$

The quadrupole source strength T_{ij} is typically approximated [7] by only the term quadratic in the velocities, the viscous stress and other terms being deemed negligible in the linear far field:

$$T_{ij} = \rho_o v_i v_j. \quad (2)$$

Here, the fluid density has been approximated by its ambient value ρ_o and the v_i are the velocity components. The solution to this equation may be expressed using the free-space Green's function for the three-dimensional wave equation as

$$p(\mathbf{x}, t) = \frac{\partial^2}{\partial x_i \partial x_j} \int d\mathbf{y} \frac{T_{ij}(\mathbf{y}, t - |\mathbf{x} - \mathbf{y}|/c_o)}{4\pi|\mathbf{x} - \mathbf{y}|} \quad (3)$$

At distances large compared to the linear extent of the source's support, this expression may be approximated by

$$p(\mathbf{x}, t) = \frac{x_i x_j}{4\pi c_o^2 |\mathbf{x}|^3} \frac{\partial^2}{\partial t^2} \int d\mathbf{y} T_{ij}(\mathbf{y}, t - |\mathbf{x} - \mathbf{y}|/c_o) \quad (4)$$

The quantity most commonly measured in acoustics experiments is the far-field mean-square pressure; it may be represented now as

$$\langle p(\mathbf{x}, t) p(\mathbf{x}, t + \tau) \rangle = \frac{x_i x_j x_k x_l}{16\pi^2 c_o^4 |\mathbf{x}|^6} \int d\mathbf{y} I_{ijkl}(\mathbf{y}, \tau) \quad (5)$$

where

$$I_{ijkl}(\mathbf{y}, \tau) = \frac{\partial^4}{\partial \tau^4} \int d\xi \langle T_{ij} T'_{kl} \rangle. \quad (6)$$

Here, T_{ij} is evaluated at position \mathbf{y} and time t , T'_{kl} is evaluated at $\mathbf{y} + \xi$ and $t + \tau$; ξ is the spatial and τ is the temporal separation of the two source strengths T_{ij} . The statistics of the source strengths are assumed to be time stationary.

The two spatial integrations in the term on the right-hand side of (5), over \mathbf{y} and ξ , have the effect of hiding information about the sources from the far-field pressure and so there is no hope of getting complete information about the noise sources without instituting a special experimental program specifically set up to probe the nature of the sources, perhaps along the lines of References [1, 2, 3]. While this is an intriguing possibility that is discussed further in the conclusion, the purpose of the present investigation is to determine what information

about the noise sources may be derived based on current standard experiments and existing experimental data.

Typically, experimental data is expressed in terms of the Fourier transform of (5) with respect to τ and so, in practice, in the following, we will be concerned with attempting to determine the noise spectrum $I_{ijkl}(\Omega)$. Furthermore, because experimental data is generally available only on some arc in the far field, parametrized by some angle, say θ , the data available to us for solving the inverse problem is a function of the two variables Ω and θ and we cannot hope to fully determine the dependence of $\int d\mathbf{y} I_{ijkl}$ on \mathbf{x} . Rather, the inverse problem as posed here has the potential for characterizing one temporal and one spatial degree of freedom of the noise source. The remainder of the noise source function must be specified *a priori*.

Additional information about the source term on the right-hand side of (5) may be deduced using the fact that $\langle T_{ij} T'_{kl} \rangle$ is essentially the fourth-order correlation of a turbulent velocity field. Thus, the characteristic length and time scales for the separation distance ξ and time τ are given by the correlation lengths and times of the turbulence. On the other hand, the variable \mathbf{y} in I_{ijkl} gives the position of the source in the flow field, and gives the dependence of the turbulence correlation on the aerodynamic data through such quantities as the kinetic energy and the dissipation. The fact that both \mathbf{y} and ξ are integrated over in determining the pressure in (5) means that, without additional experimental data, only limited information about the spatial dependence of the turbulence correlations and the aerodynamic data may be inferred when the assumptions leading up to (5) are satisfied. Consequently, we focus on determining information about the temporal dependence of the correlation (the frequency spectrum) and the directivity (the tensor nature of I_{ijkl}).

3. THE INVERSE PROBLEM VIA THE MGB CODE

The MGB approach to noise prediction [8] provides a convenient basis for an initial analysis of the inverse problem in a real jet problem. In the present work, use is made of the MGB code developed by Khavaran [7], which employs the Lilley formulation of the Lighthill acoustic analogy [5] and convective corrections as proposed by Ffowcs-Williams [6]. This code has been modified to permit alternative noise source spectra to be tested, as described in an earlier paper [9]. Here, this modification will be used to implement an iterative procedure, based on the MGB code, for the determination of a noise source spectrum that yields a given, experimentally determined, far-field noise spectrum.

The MGB code, as modified in [9], approximates the noise source spectrum in the Lighthill analogy (1) by the expression

$$I_{ijkl}(\Omega) = \hat{I}_{ijkl} I(\Omega), \quad (7)$$

where

$$I(\Omega) = \frac{2048\pi^2}{15} \rho^2 \Omega^4 \int_0^\infty dk [kQ(k)]^2 \int_{-\infty}^{+\infty} [r(k, \tau)]^2 e^{i\Omega\tau} d\tau \quad (8)$$

and

$$\hat{I}_{ijkl} = \frac{1}{16} [7(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{kj}) + 2\delta_{ij}\delta_{kl}]. \quad (9)$$

These expressions result from the standard assumption in the MGB analysis that the fourth-order correlations in I_{ijkl} may be written in terms of second-order correlations and the writing of those second-order correlations S_{ij} in a form appropriate for statistically homogeneous and isotropic turbulence [12]:

$$S_{ij}(\xi, \tau) = \int d\mathbf{k} e^{i\mathbf{k}\cdot\xi} P_{ij}(\mathbf{k}) Q(k) r(k, \tau). \quad (10)$$

Here, the tensor $P_{ij}(\mathbf{k}) = \delta_{ij} - k_i k_j / k^2$ ensures that S_{ij} is isotropic and solenoidal, $Q(k)$ is computed from the energy spectrum

$$Q(k) = \frac{1}{4\pi k^2} E(k) \quad (11)$$

and $r(k, \tau)$ describes the temporal correlation at different wavenumbers; it is normalized so that $r(k, 0) = 1$.

The discussion of the previous section indicates that we can expect to obtain via the inverse problem information about the frequency spectrum $I(\Omega)$ and the directivity \hat{I}_{ijkl} ; the various assumptions about the turbulence statistics listed above serve to eliminate the remaining uncertainties in the source strengths. Once this information is obtained, some properties of the energy spectrum $Q(k)$ and time correlation $r(k, \tau)$ may be inferred, but clearly they cannot be completely determined. Naturally, any conclusions drawn are predicated on the correctness of the assumptions leading up to this representation of the source strength.

We concentrate first on obtaining the frequency spectrum $I(\Omega)$ from experimental data; determination of the directivity \hat{I}_{ijkl} will be dealt with later. Experience in obtaining curve fits for the spectra discussed in Reference [9] suggests that an appropriate representation of the frequency spectrum is

$$I(\Omega) = \exp \left[-\sum_{i=-1}^N a_i \Omega^i \right]. \quad (12)$$

The coefficients a_i were determined using a Newton-type method, with the MGB prediction code performing the function evaluation. The difference between the target far-field pressure and that predicted with the MGB code and given values of the coefficients a_i was driven to zero by determining the “design sensitivities” with respect to the “design variables” a_i and solving for the increments to the a_i . The necessary derivatives were computed using finite differences; that is, each of the a_i was perturbed in sequence to give the columns in the sensitivity matrix. This simple scheme proved to require quite good initial guesses for the coefficients in order to converge and so a continuation scheme was implemented in which a number of intermediate target far-field pressure distributions were computed, gradually

moving towards the experimental target.

4. RESULTS

The MGB-base procedure described in the previous section was applied to the case of the Seiner jet [10] at $1550^\circ F$. The acoustic data employed in the inversion is that of Reference [10] and the aerodynamic data is from Reference [11]. The relevant plot from the latter reference is reproduced here as Figure 1. The ISAAC results with the $k - \varepsilon$ model, but without a compressibility correction, seemed to give the best agreement with the experimental aerodynamic data and are used in the present analysis.

For an initial test of the concept, data collected from a microphone at an angle of 93.3° from the jet inlet was used to construct a three-parameter approximation to the source spectrum $I(\Omega)$. (Of the coefficients in (12), a_0 , a_1 and a_2 were nonzero, the others were zero.) The results are shown in Figure 2 where it is seen that a good fit is possible and that the present results are significantly better than those of the standard MGB model for the source spectrum.

The source spectrum determined through the inverse problem is shown in Figure 3, along with the MGB frequency spectrum for comparison. The present spectrum is dramatically different: it has its peak at a much lower frequency and seems to have a fatter tail. The failure of the of the present spectrum to tend to zero as the frequency becomes small — as it must on physical grounds — is due to the absence of low-frequency experimental data in the target pressure distribution: if the pressure data had dropped to zero, then so would the result of the inverse computation.

The most important conclusion to be derived from this result is that the frequency spectrum of the noise sources seems to have a much longer time scale than that employed by the standard MGB code. As the MGB time scale is inherited from the second-order velocity

correlations through the analysis outlined in the previous section, this would seem to imply that the fourth-order correlations of the turbulent velocity field have a longer correlation time than the second-order correlations. Whether this is in fact the case or only seems to be in the present context as a result of some acoustic filtering effect akin to that described by Wilson *et al.* [13] is currently under investigation.

ACKNOWLEDGEMENT

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LIST OF FIGURES

Figure 1: 1550° F jet case, no compressibility correction. Experiment, solid line; ISAAC $k - \epsilon$ model, dash-dot-dot line; ISAAC GS ASM, long-dash line; PAB3D $k - \epsilon$ model, dotted line; PAB3D GS ASM, dash-dot line; PAB3D Girimaji ASM, dashed line. From Reference [11].

Figure 2: SPL at a microphone angle of 93.3° for the 1550°F jet. Experiment, solid line. Khavaran MGB code, dashed line. Present computation, using source spectrum from inverse problem, dotted line.

Figure 3: $I(\Omega)/I_{\max}$ for the the noise spectrum found by solving the inverse problem (solid line) and for the MGB noise spectrum (dashed line).

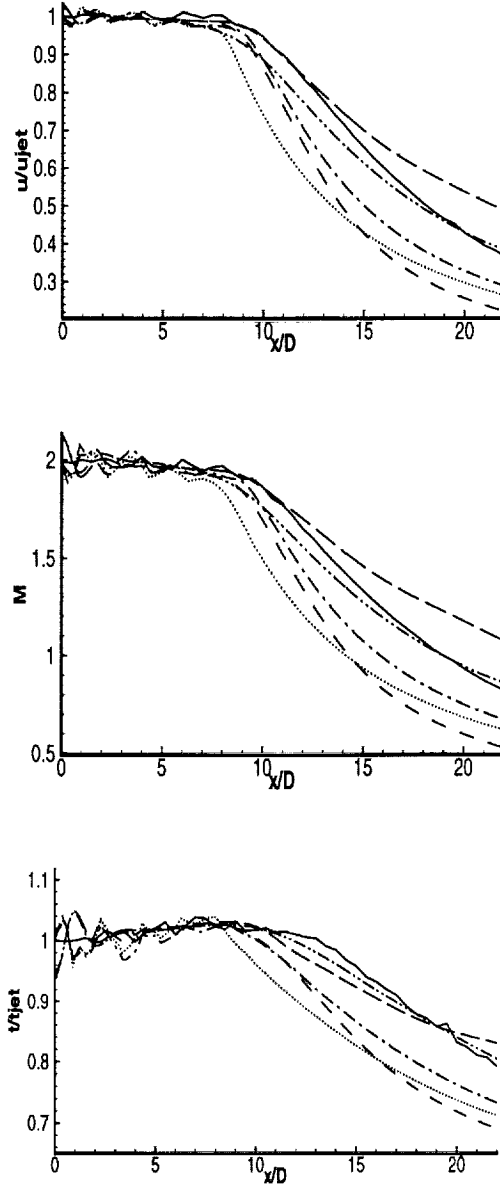


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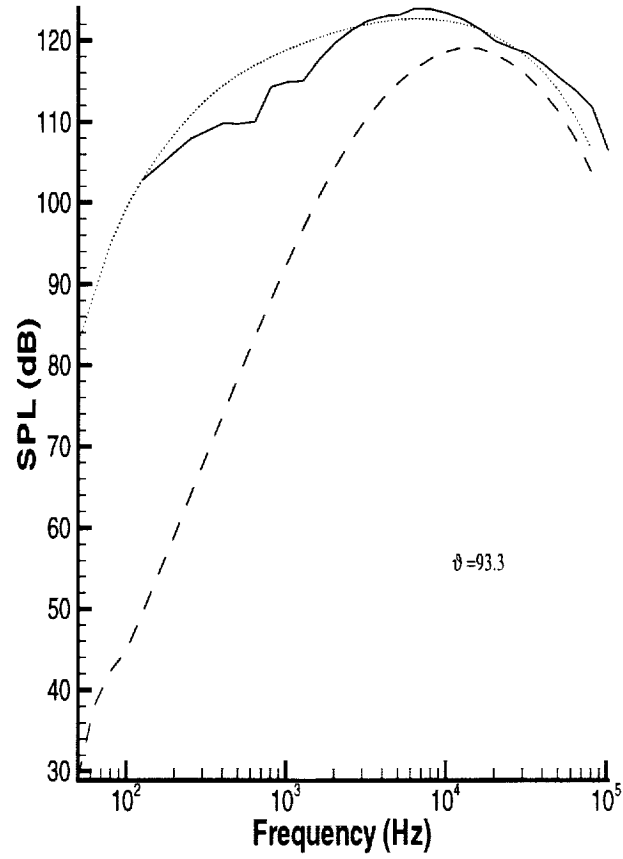


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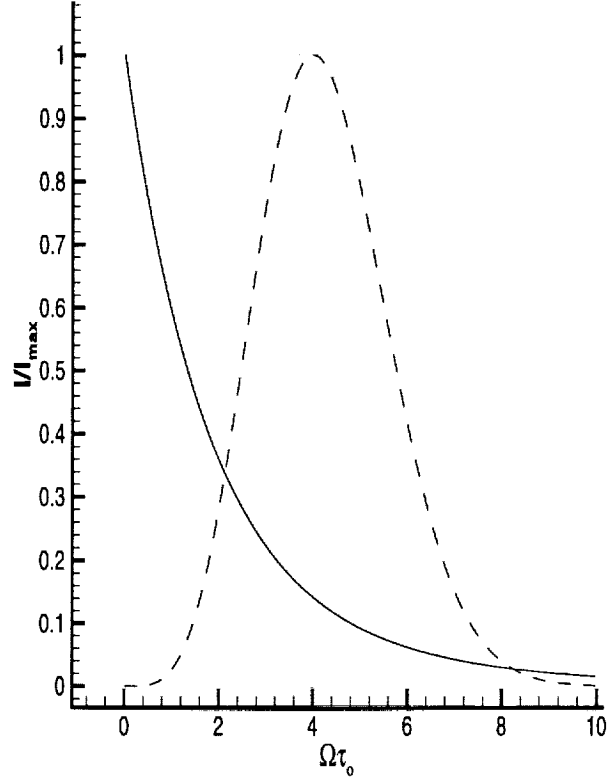


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